

HMC 2007 FIRST ROUND QUESTIONS WITH ANSWERS

1. "Two hundred and forty-seven thousand" is

- a) 204700 b) 204070 c) 247000 d) 240700

Answer: 247000

2. $0 + 1 + 3 + 5 = ?$

- a) 8 b) 9 c) 10 d) 11 e) 12

Answer: 9

3. Find the next number in the following number sequence.

2, 7, 13, 20, 28, ...

- a) 33 b) 35 c) 37 d) 39 e) 43

Solution: another look at the sequence is 2, 2+5, 7+6, 13+7, 20+8. Therefore we can say the next term is 28+9=37 with ease.

4. What arithmetic symbol can we place between 2 and 3 to make a number greater than 2 but less than 3?

- a) + b) - c) \times d) \div e) a decimal point

Answer: a decimal point

5. If p and q are numbers and $p \odot q = \frac{p+q}{2}$, find $1 \odot 3 = ?$

- a) 2 b) 3 c) 5 d) 6 e) None of these

Solution: $1 \odot 3 = \frac{1+3}{2} = 2$

6. A *palindromic* number is any number which has the same value when read from either direction. Therefore 2002 is a palindromic year. How long will it take to have the next palindromic year?

- a) 10 b) 100 c) 110 d) 1001 e) None of these

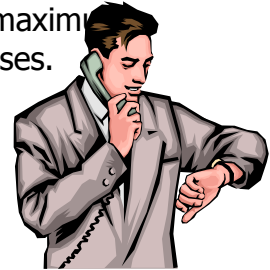
Solution: The next smallest palindromic number is 2112. Therefore it will take a hundred and ten years.



7. How many buses would be required to transport 329 students, if each bus holds a maximum of 47 students?

- a) 5 b) 6 c) 7 d) 8 e) 9

Solution: all we need to do here is to divide the number of students to the maximum number of students a bus can hold. $329 \div 47 = 7$. This means we need 7 buses.



8. Find the angle between the hands of a clock at 5 o'clock.

- a) 90° b) 120° c) 130° d) 150° e) 170°

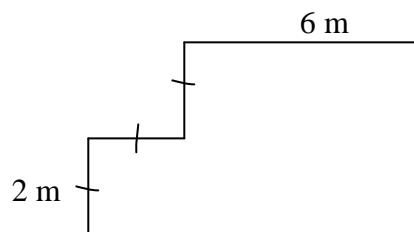
Solution: at 5 o'clock the minute hand is on the number 12 and the hour hand is on the number 5. Between two consecutive numbers on the clock the angle is 30 degrees. Therefore $30 \times 5 = 150$ degrees.

9. A man works for 10 days. On the first day he is paid R2. On the second day, R4. On the third day, R8. On each successive day, his pay is doubled. How much is he paid altogether for the 10 days?

- a) 1023 b) 1999 c) 2000 d) 2046 e) 2048

Answer: 2046

10. Find the perimeter of the following figure.
(The distance around the figure)



- a) 8m b) 18m c) 20m d) 22m e) 24m

Solution: if you investigate the figure carefully you will observe that to find its perimeter is equal to find the perimeter of a rectangle with dimensions 6 by 4. (Check!) Therefore its perimeter is $2 \times (6 + 4) = 20\text{m}$.

11. How thick is a single sheet of ordinary paper, approximately?

- a) 10mm b) 5mm c) 1mm d) 0.5mm e) 0.1mm



Solution: 1 mm is so thick for an ordinary paper. But 0.1 mm is a reasonable approximation.



12. What is the time 34 hours after 2 o'clock?

- a) 10:00 b) 11:00 c) 12:00 d) 13:00 e) 14:00

Solution: we know any multiple of 12 hours after a particular time will be exactly the same because of the cyclic nature of the clock. That means 24 hours after 2 o'clock is still 2 o'clock (*pm* and *am* is out of discussion in this question). Since $34 = 24 + 10$, 34 hours after 2 o'clock means 10 hours after 2 o'clock which is 12 o'clock.

13. How many times larger is the first 8 than the second in 86482?

- a) 10 b) 80 c) 100 d) 1000 e) 7920

Solution: the first 8 in the given 5-digit number has the place value of 80000 and the next one's is 80. Therefore the difference between them, 7920, is the answer.

14. John ate 2 more nectarines each day than on the previous day, over a five-day period. If he ate 45 altogether, how many did he eat on the first day?

- a) 3 b) 5 c) 7 d) 8 e) 9

Solution: let John eat n nectarines on the first day. Then on the next day he will eat $n+2$ nectarines, then $n+4$, then $n+6$, then on the fifth day he will have eaten $n+8$ nectarines. The total amount of consumed nectarines is given as 45. Thus we have the following equation:

$$\begin{aligned} n + n+2 + n+4 + n+6 + n+8 &= 45 \\ 5n+20 &= 45 \\ 5n &= 25 \\ n &= 5 \end{aligned}$$

15. What fraction of the day is 180 minutes?

- a) $\frac{3}{4}$ b) $\frac{1}{8}$ c) $\frac{1}{4}$ d) $\frac{1}{2}$ e) None of these



16. Find the smallest natural number (greater than 1) which when divided by 2, 3, 4 and 5 respectively leaves a remainder of 1 on every occasion.

- a) 41 b) 51 c) 61 d) 71 e) 81

Solution: if we multiply the numbers given as divisors, the resulting number will be divisible by each one since it contains each of them as its factors. Therefore $2 \times 3 \times 4 \times 5 = 120$. There is no remainder for now.

17. Donald Duck can eat 2 pizzas in 3 minutes, while Goofy can eat 3 pizzas in 2 minutes. At these rates, how many pizzas can they eat together in an hour?

- a) 54 b) 96 c) 130 d) 216 e) 250



Answer: 130

18. If $a + b = 26$ and $a - b = 12$, then find the value of $a \times b$.

- a) 123 b) 130 c) 133 d) 135 e) 143

Solution:

If we add the left hand sides of the given relations we have the following arguments:

$$\begin{aligned} (a + b) + (a - b) &= 26 + 12 && \{\text{remove the brackets}\} \\ a + b + a - b &= 38 && \{\text{rearrange the letters}\} \\ a + a + b - b &= 38 && \{\text{collect the like terms}\} \\ 2a + 0 &= 38 && \{\text{simplify}\} \\ 2a &= 38 && \{\text{divide both sides by 2}\} \\ a &= 19 \end{aligned}$$

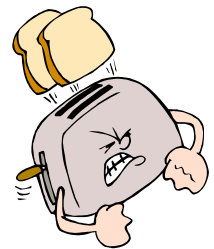
Then we can find the value of b immediately since

$$\begin{aligned} a + b &= 26 && \{\text{given}\} \\ 19 + b &= 26 && \{\text{substitute the value of } a\} \\ b &= 26 - 19 \\ b &= 7 \end{aligned}$$

Therefore $a \times b = 19 \times 7 = 133$

19. An old fashioned toaster can toast one side of up to 4 slices of bread in one minute. What is the least time required to toast both sides of 9 slices?

- a) 4 b) 5 c) 6 d) 7 e) None of these



Answer: 5

9 slices have $18 = 4 \cdot 4 + 2$ sides, thus require at least 5 minutes. One way to toast all 18 sides in 5 minutes: toast 1, 2, 3, 4, then 3, 4, 5, 6, then 1, 2, 5, 6, then 7, 8, 9, then 7, 8, 9.

20. Dumb-bells weigh 2, 3 or 4 kg. The total weight of a pile of dumbbells is 80 kg. The number of dumb-bells in the pile that weigh 3 kg can **not** be

- a) 2 b) 3 c) 4 d) 6 e) 10

Answer: 3

There cannot be an odd number of dumbbells that weigh 30 kg, else the total weight would be an odd multiple of 10.

21. In any calendar year, what would be the maximum number of Saturdays that fall on the 9th day of the month?

- a) 1 b) 2 c) 3 d) 4 e) 5



Solution:

We give here an analysis of years having 365 days; the analysis for 366 day years is similar. First, let us call Sunday a *Type 0* day, Monday a *Type 1* day, Tuesday a *Type 2* day, ..., Saturday a *Type 6* day. If January 13 is a *Type 0* day, then February 13 is a *Type 3* day since it is 31 or 3 + 28 days later, March 13 is a *Type 3*, April 13 is a *Type 6*, May 13 is a *Type 1*, ..., December 13 is a *Type 5* day. The entire list of types from January 13 to December 13 is

0, 3, 3, 6, 1, 4, 6, 2, 5, 0, 3, 5

There are two Friday the thirteenths, since Friday is of *Type 5*. The analysis thus far has been on the assumption that January 13 is a Sunday. The easiest way to proceed is to vary what is meant by *Type 0* to be Monday, then Friday is of *Type 4*, and the list above shows that in such a year there is only one Friday the 13th. Thus the list reveals the answer to the problem by consideration of all seven interpretations of what is meant by *Type 0*. For a 366 day year, the corresponding list is

0, 3, 4, 0, 2, 5, 0, 3, 6, 1, 4, 6

22. How many positive 5-digit integers have the result of 2000 when their digits are multiplied?

- a) 15 b) 20 c) 25 d) 30 e) 100

Solution: Let a five-digit integer be written **abcde** where $0 \leq a, b, c, d, e \leq 9$, and $a \neq 0$. We have $a \times b \times c \times d \times e = 2000 = 2^4 \times 5^3$. Thus three of the digits have to be 5. The remaining two digits must be single digit factors of 16.

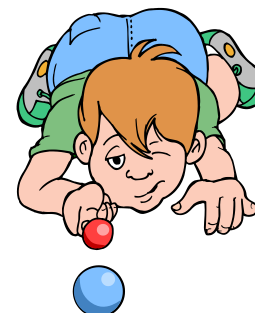
There are thus two possibilities, 4 and 4 or 2 and 8. In the first case, three

5s and two 4s may be ordered in $\frac{5!}{3! \times 2!} = 10$ ways. In the second case, three

5s, a 2 and an 8 may be ordered in $\frac{5!}{3! \times 1!} = 20$ ways. Thus the total is 30.

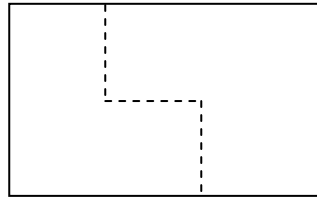
23. There are 37 red, green, blue, and yellow marbles in a bag. There are 3 more red marbles than green marbles, 2 more red than blue, and 4 more yellow than blue. How many green marbles are there?

- a) 5 b) 7 c) 9 d) 11 e) 12



Answer: 7

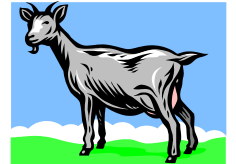
24. You are given a piece of wood measuring 8 cm by 18 cm. The wood is cut into two pieces roughly as shown below, and the pieces are then rearranged to make a square.



Which one of the following gives the area of the square?

- a) 10×10 b) 11×11 c) 12×12 d) 13×13 e) 14×14
25. Each of my three uncles owns some goats. They have different numbers of goats, each 3 or more, and multiplying the three numbers together gives the product 108. What is the total number of goats?

- a) 15 b) 16 c) 17 d) 18 e) 19



Solution.

The prime factorisation of 108 is $2^3 \times 3^3$. The five 2's and 3's in these factors must be split up between the three numbers, so one number receives only one of them. Since 2 is too small, this number must be 3. The remaining two 2's and two 3's must be split between the remaining two numbers, with each receiving two primes. Since they are different numbers, this can only be done by one receiving two 2's and the other two 3's. That is, the second number is 22 and the third is 32. So the three numbers are 3, 4 and 9.

26. Joe had a number of jellybeans. Half of them were red, one third of them were blue and the rest were green. Amanda took one third of the red ones and half of the green ones, and then Cassandra took half of the remaining red ones and a quarter of the blue ones. Exactly 54 jellybeans remained in total. How many were there originally?

- a) 75 b) 86 c) 97 d) 108 e) 209

Solution.

If T was the total number of jellybeans originally, then $\frac{1}{2}T$ were red, $\frac{1}{3}T$ were blue and

$\frac{1}{6}T$ were green. Amanda took $\frac{1}{3} \times \frac{1}{2}T = \frac{1}{6}T$ red ones, and

$\frac{1}{2} \times \frac{1}{6}T = \frac{1}{12}T$ green ones. This left $\frac{1}{2}T - \frac{1}{6}T = \frac{1}{3}T$ red ones, of which Cassandra took

half, leaving $\frac{1}{6}T$ red. Cassandra also *left behind* $\frac{3}{4} \times \frac{1}{3}T = \frac{1}{4}T$ blue ones, and there were

$\frac{1}{12}T$ green remaining. So the total number remaining was $\frac{1}{6}T + \frac{1}{4}T + \frac{1}{12}T = \frac{1}{2}T$. This was 54, so T was 108.

27. Five girls A, B, C, D, E sit on 2 chairs and 3 stools, each seating exactly one girl. Who sits on the chairs if A and B sit on the same type of seat, B and D sit on a different type, D and E sit on a different type?

- a) CD b) AD c) BE d) BC e) AB

Answer: CD

A, B and E must sit on the same type of seat, which must then be stools. So C and D sit on chairs.

28. 294 digits are used to number the pages of a book. How many pages does the book have?

- a) 127 b) 134 c) 140 d) 146 e) 151



29. Berries are 90% water and 10% pulp, measured by weight, when freshly picked. If the berries sit for a week, part of the water evaporates and the berries become 80% water and 20% pulp. If 20 kg of freshly picked berries sit for a week, how much will they weigh?

- a) 8 kg b) 10 kg c) 12 kg d) 15 kg e) 18 kg

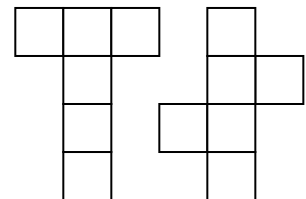


Answer: b

Originally the pulp in the berries weighs 2 kg and is 1/10th of the total weight. After 1 week the pulp (2 kg) is 1/5th of the total weight. Hence the weight is 10 kg.

30. If you fold the nets given alongside properly you will have a cube. How many different nets are there that fold into cubes?

- a) 5 b) 7 c) 10 d) 11 e) 12



Solution: The nets are:

